

Quadratic Equations

A quadratic equation in the variable x is an equation of the form $ax^2 + bx + c = 0$, where a, b, c are real numbers, $a \neq 0$.

Roots of a Quadratic Equation:

➤ A real number α is called a root of the quadratic equation

$$ax^2 + bx + c = 0, a \neq 0 \text{ if}$$

$$a\alpha^2 + b\alpha + c = 0.$$

➤ $x = \alpha$ is a solution of the quadratic equation, or α satisfies the quadratic equation.

➤ The zeroes of the quadratic polynomial $ax^2 + bx + c$ and the roots of the quadratic equation $ax^2 + bx + c = 0$ are the same.

Solution of Quadratic Equation by Factorisation:

➤ To factorise quadratic polynomials the middle term is split.

➤ By factorizing the equation into linear factors and equating each factor to zero the roots are determined.

Quadratic Equations - Method of Squares

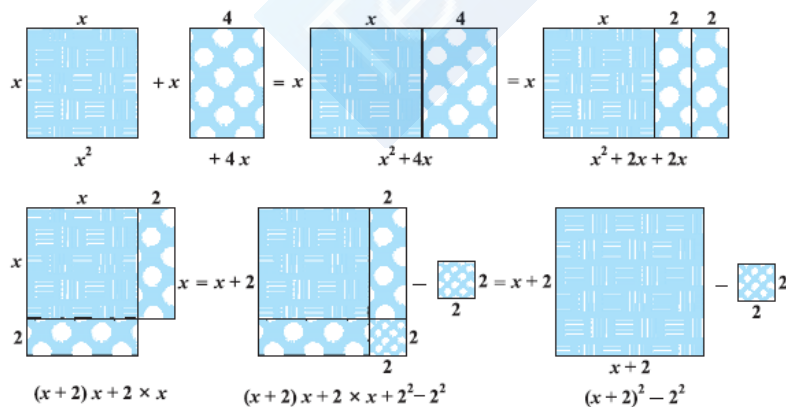
Solution of Quadratic Equation by method of Squares

➤ We can convert any quadratic equation to the form

$$(x + a)^2 - b = 0$$

$$x^2 + 4x \text{ is being converted to}$$

$$(x + 2)^2 - 4 = (x + 2)^2 - 2^2$$



The process is as follows:

$$\begin{aligned} & (x^2 + -x) - \\ = & \\ & (x - 2) \\ & (x - 2) \\ & (x + 2) x + (x + 2) \times \\ & (x + 2) (x + 2) - \\ & (x - 2) \end{aligned}$$

So, $x^2 + 4x - 5 = (x + 2)^2 - 4 - 5 = (x + 2)$

So, $x^2 + 4x - 5 = 0$ can be written as $(x + 2)$ by this process of completing the square. This is known as the **method of completing the square**.

Solution of Quadratic Equation by using Formula.

The formula is as follows:

The roots of $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

If

Thus, if $b^2 - 4ac > 0$ then the roots of the quadratic equation are given by

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

This formula for finding the roots of a quadratic equation is known as the Quadratic formula.

Nature of Roots

We know that roots of the equation $ax^2 + bx + c = 0$ are

$$\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Where $b^2 - 4ac$ is known as discriminant.

Nature of roots based on the discriminant value

1. If $b^2 - 4ac = 0$ then the roots are real and equal.
2. If $b^2 - 4ac > 0$ then the roots are real and distinct (unequal)
3. If $b^2 - 4ac < 0$ then the roots are imaginary (not real)